

Statistics Solutions

Probability density functions

1. If $p(x) = x/2$ for x ranging from 0 to 2, and $p(x) = 0$ otherwise, what is the mean of this density?

Answer: $4/3$. This is calculated by $E(x) = \int_0^2 x * f(x) = \int_0^2 \frac{1}{2}x^2 = [\frac{1}{6}x^3]_0^2 = \frac{4}{3}$

2. If $p(x) = 2x/9$ for $0 < x < 3$, and $p(x) = 0$ otherwise, what is $P(0 < x < 1)$?

Answer: $1/9$

3. What is the variance of a uniform variable with range a to b .

Answer: Between a and b the density is $1/(b - a)$, so the second moment is

$$E(x^2) = \int_a^b x^2(1/(b - a))dx = \frac{b^3 - a^3}{3(b - a)} = \frac{(b^2 + ab + a^2)(b - a)}{3(b - a)} = \frac{b^2 + ab + a^2}{3}.$$

$$\sigma_x^2 = E(x^2) - \mu_x^2 = \frac{b^2 + ab + a^2}{3} - \left(\frac{b + a}{2}\right)^2 = \frac{b^2 - 2ab + a^2}{12} = \frac{(b - a)^2}{12}$$

4. If scores are normally distributed with a mean of 35 and a standard deviation of 10, what percent of the scores is:

- (a) greater than 34?

Answer: This is a normal distribution of $N(35, 100)$. We need to compute $P(X > 34) = P(Z > \frac{34-35}{10}) = P(Z > -0.1) = \frac{1}{2} + P(-0.1 < Z < 0)$. After lookup in the table we get $\frac{1}{2} + 0.0398 = 0.5398$.

- (b) smaller than 42?

$P(X < 42) = P(Z < \frac{42-35}{10}) = P(Z < 0.7) = \frac{1}{2} + P(0 < Z < 0.7) = \frac{1}{2} + 0.2580 = 0.7580$.

- (c) between 28 and 34?

$P(28 < X < 34) = P(\frac{28-35}{10} < Z < \frac{34-35}{10}) = P(-0.7 < Z < -0.1) = P(-0.7 < Z < 0) - P(-0.1 < Z < 0) = 0.2580 - 0.0398 = 0.2182$.

5. The life of Sunshine DVD players is normally distributed with a mean of 4.1 years and a standard deviation of 1.3 years. A DVD player is guaranteed for 3 years. We are interested in the length of time a DVD player lasts.

- (a) Find the probability that a DVD player will break down during the guarantee period.

We have a normal distribution of $N(4.1, 1.3^2)$. We need to calculate $P(X < 3)$. $P(X < 3) = P(Z < \frac{3-4.1}{1.3}) = P(Z < -0.85) = \frac{1}{2} - P(-0.85 < Z < 0) = 0.5 - 0.3023 = 0.1977$.

- (b) Find the probability that a DVD player will last between 2.8 and 6 years.

$P(2.8 < X < 6) = P(-1 < Z < 1.46) = 0.3413 + 0.4279 = 0.7692$.

- (c) Find the 70th percentile of the distribution for the time a DVD player lasts.

$P(Z < \frac{a-4.1}{1.3}) = 0.7 \Rightarrow P(0 < Z < \frac{a-4.1}{1.3}) = 0.2 \Rightarrow \frac{a-4.1}{1.3} \approx 0.525 \Rightarrow a \approx 4.789$.